Spiral length design

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Abstract:

On the circular curve, lateral acceleration affects comfort. On the spiral, both lateral acceleration and jerk affect comfort. The determination of maximum curving speed on the basis of lateral acceleration only is not critical because the lateral acceleration is proportional to the square of speed whereas jerk is proportional to cubic power of speed. The maximum speed on a curve is thus significantly restricted by the spiral length. In this paper, spiral length is formulated on the basis of both lateral acceleration and jerk. The proposed formula is applied on curves of various curvature and validated against an earlier formula by the author. On the basis of current choice of values of lateral acceleration and jerk, a relation between jerk and lateral acceleration is given that can be used to determine the acceptable value of jerk for a given lateral acceleration. A single criterion is proposed to decide the requirement of a spiral curve.

Keywords: *spiral length, unbalanced superelevation, actual super-elevation, jerk, lateral acceleration.*

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1. Introduction

In an earlier paper (Hasan 2010) by the author the spiral length is given by:

$$L = 0.0018V\frac{(Ea + Eu)}{J} \tag{1}$$

On circular curve, theoretically lateral jerk is zero since the lateral acceleration is constant. On the other hand, the lateral acceleration is variable and jerk is constant, equivalent to design value, on the spiral. Eq. (1) considers speed, super-elevation, and jerk to compute the spiral length. It does not consider lateral acceleration explicitly. The comfort criterion is usually given by lateral acceleration. In this paper, a formula is derived that includes lateral acceleration explicitly in addition to speed, super-elevation, and jerk. To validate, the formula is applied and results are compared with the results given by Eq. (1). Usually lateral acceleration is given. A relation is given to determine the value of jerk for any given lateral acceleration. The determination of maximum curving speed on the basis of lateral acceleration only is not critical because the lateral acceleration is proportional to the square of speed whereas jerk is proportional to cubic power of speed. The maximum speed on a curve is thus significantly restricted by the spiral length. The suggested spiral length equation is useful to determine the maximum curving speed on a curve as it contains both terms - lateral acceleration and jerk.

2. Derivation of spiral length formula

In this section, jerk and lateral acceleration refer to axle based values. Jerk is defined as the rate of change of lateral acceleration. Mathematically

$$J = \frac{a}{t}$$

$$J = \frac{\frac{v^2}{R}}{\frac{L}{v}} = \frac{v^3}{RL} \tag{2}$$

$$J = \frac{\frac{v^2}{R}}{\frac{L}{v}} = \frac{v^3}{RL} \tag{3}$$

The well known equilibrium super-elevation equation is given by:

$$Eq = Ea + Eu = 153\frac{v^2}{R}$$

$$Ea = 153\frac{v^2}{R} - Eu \tag{4}$$

Dividing both sides of Eq. (4) by L,

$$\frac{Ea}{L} = \frac{153\frac{v^2}{R} - Eu}{L} = \frac{153v^2}{RL} - \frac{Eu}{L} \tag{5}$$

Replacing L on right side of Eq. (5) by Eq. (3):

$$\frac{Ea}{L} = 153 * \frac{J}{v} - Eu * \frac{JR}{v^3}$$

$$\frac{Ea}{L} = 153 * \frac{J}{v} - Eu * \frac{J}{v * \left(\frac{v^2}{R}\right)}$$

$$\frac{Ea}{L} = 153 * \frac{J}{v} - \frac{Eu}{v} * \left(\frac{J}{a}\right)$$

$$\frac{Ea}{L} = \frac{153 * J * a - Eu * J}{va}$$

$$L = \frac{Ea * v}{153 * J - Eu * \left(\frac{J}{a}\right)}$$

Table 1. Spiral length by Eq. (1) and Eq. (4)

							Comfort limit		Spiral Length, L(m)	
V	R	Curvature	Eq	Ea	Eu	Eu/Ea	Jerk, J (g/s)	Lat. acc^n, a (g)	By Eq. (1)	By Eq. (4)
35	120	Super-sharp	120	70	50	0.72	0.03	0.1	26	23
45	175	Very sharp	137	79	57				38	36
60	300	Sharp	142	82	59				52	50
85	600	Mild	142	83	59				74	72
100	800	Mild	148	86	62				90	90

Changing the unit of speed from m/s to km/h,

$$L = \frac{VEa}{551*J - 3.6*Eu*\left(\frac{J}{a}\right)} \tag{5}$$

3. Validation

The Eq. (6) is validated by application only. The Eq. (1) and Eq. (6) are applied for three pairs of axle based lateral acceleration and jerk on different curvatures in the following three Table (1) ~ (3):

Usually the spiral length is rounded to nearest 5m. In the above tables spiral lengths are not rounded. Thus it appears from the above three tables that the Eq. (6) gives the same spiral length as the Eq. (1) which is already an accepted formula. Theoretically Eq. (6) is a better expression as it contains both lateral acceleration and jerk.

In fact Eq. (1) and Eq. (6) are same. From Eq. (6), Eq. (1) can be deduced. This is shown below:

$$L = \frac{Ea*v}{153*J - Eu*\left(\frac{J}{a}\right)} = \frac{Ea*v}{153*J - \frac{Eu}{t}} =$$

$$\frac{Ea*v*t}{153*J - Eu} = \frac{Ea*L}{153*J*t - Eu}$$

Taking out L from both sides,

$$1 = \frac{Ea}{153*J*t - Eu}$$

$$153*J*t = Ea + Eu$$

$$t = \frac{Ea + Eu}{152*J}$$

$$L = \frac{Vt}{3.6} = \frac{V(Ea + Eu)}{3.6*152*J} = 0.0018V(Ea + Eu)$$

Under equilibrium (balanced) speed, both Eq. (1) and Eq. (6) give a superelevation gradient of 1 in 6V [V(kmph)] under a jerk of 0.03 g/s. Since trains always run with some unbalanced superelevation, the minimum superelevation gradient would be flatter than 1 in 6V. On NS steeper transition gradients of up to 1: 8V are allowed with a minimum of 1 in 600 at speeds lower than 80 km/h (Esveld 2001).

Table 2. Spiral length by Eq. (1) and Eq. (4)

							Comfort limit		Spiral Length, L(m)	
V	R	Curvature	Eq	Ea	Eu	Eu/Ea	,Jerk J (g/s)	.Lat ,acc^n a (g)	By Eq. (1)	By Eq. (4)
35	120	Super-sharp	120	70	50	0.72	0.04	0.1	19	17
45	175	Very sharp	137	79	57				28	27
60	300	Sharp	142	82	59				39	38
85	600	Mild	142	83	59				55	54
100	800	Mild	148	86	62				68	67

Table 3. Spiral length by Eq. (1) and Eq. (4)

							Comfort limit		Spiral Length, L(m)	
V	R	Curvature	Eq	Ea	Eu	Eu/Ea	,Jerk J (g/s)	.Lat ,acc^n a (g)	By Eq. (1)	By Eq. (4)
220	2500	Super-sharp	228	133	96	0.72	0.10	0.15	92	94
240	3000	Very sharp	227	132	95				100	101
255	3500	Sharp	219	127	92				103	101
270	4000	Mild	215	125	90				107	104

4. Relation between jerk and lateral acceleration

Usually a comfort value of lateral acceleration is suggested without any mention of corresponding jerk as comfort criteria.

As per UIC Code, the comfort limit is 1.0 to 1.5 m/s^2 range (=0.1g to 0.153g) (UIC Code 1989).

According to Esveld (2001), the lateral acceleration must in all cases remain below 1.5 m/s^2, and preferably below 1 m/s^2. In North America the comfort limit is usually set at 0.1g. Jerk is also an important parameter which affects ride comfort.

To use Eq. (6), one needs both lateral acceleration and jerk value. Using the values of lateral acceleration and jerk used in current practice, a relation between lateral acceleration and jerk is established hereafter to facilitate the use of Eq. (6). Generally, the limit of comfort refers to values measured in the carbody's reference system. These are usually measured on the car's floor, on top of the leading and trailing bogie pivot, at the connection between bogie and carbody. In literature (TCRP 2012) and here, the values are applied on axle basis. Usually the perceived values on floor are reduced from the axle based values.

Thus, the application of codified comfort values on axle basis is conservative. In Tables 1~3, the values of pair of lateral acceleration and jerk e.g. 0.1g and 0.03g/s, 0.1g and 0.04 g/s, and 0.15g and 0.1g/s are taken from TCRP (2012). TCRP does not clear the basis of the choice of the values of the pair.

The spiral lengths calculated in Tables 1~3 indicate that the suggested choice by TCRP works. Jerk and lateral acceleration are linearly related (cf. Eq. (2)). Thus, a relation between lateral acceleration and jerk is established by interpolation between values of two pairs e.g. 0.1g and 0.03g/s, and 0.15g and 0.1 g/s. The relation stands as:

$$J = 0.03g/s + 1.4(a - 0.1g) \tag{7}$$

in which

a = lateral acceleration in terms of 'g' e.g. 0.1g, 0.15g etc.

As per Eq. (7), for a jerk of 0.04 g/s, theoretically lateral acceleration should be 0.107g instead of 0.1g in Table 2. For a lateral acceleration of 0.2g, jerk should be 0.17g/s. Eq. (7) would help to use Eq. (6) if jerk values are not given. Eq. (7) would help to judge the given value of jerk for a given lateral acceleration.

5. Utility of the suggested formula

The formula has two other applications other than determining the spiral length. They are:

First application: Curving speed model

The formula may be manipulated to model curving speed.

Second application: Requirement of spiral curve

In the literature, authors often put some criteria where spiral curves are not required. As for example, Esveld (2001) says – spiral curves are not used if:

- The curve radius is > 3000 m;
- A calculation shows that no superelevation is necessary;
- Between two adjacent curves in the same direction the discontinuity in acceleration remains limited to 0.2 ~ 0.3 m/s^2.

In the literature, authors also put some guide line on installation of superelevation. Esveld (2001) says if the

Table 4. Testing of single criterion for the requirement of spiral curve

V (kmph	R (m)	Eq (mm)	Ea (mm)	Eu (mm)	a (m/s^2)		L (m)	Requirement of SC
20	250	19	11	8	0.12		5> 1.43	.Not reqd
50	1200	25	14	10	0.16		5> 4.73	.Not reqd
75	3000	22	13	9	0.14		5< 6.34	.Absolute min length reqd
80	3200	24	14	10	0.15		5< 7.25	.Absolute min length reqd

calculated superelevation is less than 20 mm it can be disregarded. If the *calculated superelevation* is 25 mm or less, actual superelevation is not usually installed (TCRP 2012). In the preceding statements, the *calculated superelevation* means equilibrium superelevation.

Thus, actual superelevation should be less than 20 mm to avoid installation. According to proportion between unbalanced and actual superelevation suggested by me (Hasan 2011), the actual superelevation comes out to be 12 mm (=20/1.72) or less to avoid installation.

In USA, there are many instances of installation of half inch superelevation. With a minimum superelevation gradient of 1 in 600 (Esveld 2001), 12 mm of actual superelevation leads to a spiral length of 7.2 m. Thus non-installation of actual superelevation may not call for non-installation of spiral; it needs to be confirmed from the required length of spiral curve.

Usually, the spiral length is rounded to the nearest 5 m. Thus, I would like to propose a single criterion that if the calculated spiral length is less than 5 m, then it is not necessary to install spiral curves.

In all other cases at least absolute minimum length of spiral curve shall be installed. The proposed criterion for requirement of spiral curve is analyzed in Table 4.

6. Future research

In this paper, ride comfort is estimated by quasi-static formula, which has limitations. The formula considers only vertical stiffness of suspension via suspension factor; in fact, ride comfort is affected by many other vehicle parameters. It would be very expensive to measure ride comfort by a simple accelerometer or track geometry car on a curve with the proposed spiral length. Currently, there are sophisticated, validated computer software programs that estimate ride comfort. These programs also can predict derailment safety by computing lateral load/ vertical load (L/V) ratios. Use of any suitable software is recommended to estimate ride comfort and safety. This would certainly help to validate the work and increase confidence in it.

7. Conclusion

The spiral length derived is different in its form from all current equations. The equation may be manipulated to model curving speed. The spiral length is given by

$$L = \frac{VEa}{551 * J - 3.6 * Eu * \left(\frac{J}{a}\right)}$$

For a given lateral acceleration, jerk is to be determined by the following relation:

$$J = 0.03g / s + 1.4(a - 0.1g)$$

Length is suggested as a criterion for the requirement of installation of spiral curve. If the calculated spiral length is less than 5 m, then the spiral is not required.

8. Notations

a = axle based lateral acceleration in m/s^2;
Ea = actual super-elevation in mm;
Eq = equilibrium super-elevation in mm;
Eu = unbalanced super-elevation in mm;
J = axle based jerk in m/s3;
L = spiral length in m;
R = radius of curve in m;
t = time to traverse spiral in s;
V = speed in km/h;
v = speed in m/s.

9. References

- Esveld, C. (2001). *Modern railway technology*, MRT-Productions, the Netherlands: 38, 39, 61.

- Hasan, N. (2010). "Spiral length design," *Proc., 2010 ASME/ASCE/IEEE Joint Rail Conference*, Urbana-Champaign, IL

- Hasan, N (2011). "Passenger track curve design criteria: Comfort criteria, equivalent comfort criteria, and applications," *Proc., 2011 ASME/ASCE/IEEE Joint Rail Conference*, Pueblo, CO

- International Union of Railways (UIC) Code 703. (1989). "Layout characteristics for lines used by fast passenger trains." *International Union of Railways*, 2nd edition, Paris, France: 8, 9.

- Transit Cooperative Research Program (TCRP). (2012). "Track design handbook for light rail transit." *Report 155*, National Academy Press, Washington, DC.