Solving railway line planning problems using a decomposition algorithm

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Abstract:
The passenger line planning is a process of strategic long-term decision-making problem in the field of railway passenger planning. A line is a route between starting point and destination with certain stops, which has a certain frequency of train schedule. A new solution based on decomposition algorithm has been proposed in this paper, which is defined in a master problem and two sub problems. Since the solution provided by decomposition algorithm is not of the integer number type, a heuristic algorithm has been proposed for converting the results to integer numbers. The objective function for the line planning problem in this article is to maximize the number of direct passengers. Results on the proposed solution method, for the test problems, are compared to those of solutions generated via CPLEX software. Results show that the proposed solution method has high performance and accuracy. As a case study, optimized passenger railway lines in Iranian railway have been determined using the proposed algorithm.

Keywords: *railway transportation planning, passenger railway planning, line planning problem, decomposition algorithm.*

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1. Introduction

The line-planning problem (LLP) or passenger route planning is a strategic and high-ranking scheduling task in passenger transportation plans of railway sector. In railway literature, line is a route between the point of start and destination with certain stops, which has a certain frequency of train schedule. Different trains move in a certain route with predetermined starting points and destinations and different departure and arrival times. The goal of this problem is to select the optimized lines among all the proposed lines, which provide the maximum available service to the customers who are the passengers in this case. The provided service can be identified by a variety of indices. The number of direct passengers is one of the most important indices. Passengers usually tend to travel directly from starting point to the destination. Direct passengers in line planning are the term used for passengers who do not have to change their train between their starting point and destination (Caprara et al., 2007).

The inputs required for line planning problem include existing infrastructures (stations and the railway between them) and travel demand between each pair of the start-point and destination. Therefore, the demand is defined as a matrix of starting point–destination. If the railway network is presented by G = (V, E), then V illustrates the stations, and E represents the routes between two consecutive stations - or the arcs in the network. In the line-planning problem, only the stations for which the number of passengers is higher than a certain minimum are considered. Therefore, the stations with freight transport or cross sectional nature and the stations whose passengers are fewer than the threshold are not considered. The set V is constituted of n stations or nodes. Input demand matrix is of the n×n dimension. Note that the one-way travel demand travel is considered as the returned demand and is defined as demand for travel and return route. The reason for this assumption is cyclical usage of the rolling stock. That means the train returns in the opposite direction of the travel. Therefore, the frequency in the travel route should be equal to the frequency of return route. That is the reason the maximum demand for travel or return route is to be considered as demand for the pair of stations. The passengers at the starting point may have the choice of several routes to reach the destination, but in all the line planning models it is assumed that they would choose the shortest rail route to get to their destinations. In addition, the routes can such be chosen that they stop at all the stations, or stop on the certain stops at the same route according to the type of train they use. Note that the travel demand between each pair of the stations would increase if the selected lines lead to the higher satisfaction of the customers or passengers. Moreover, this would guarantee the increase and sustainability of organization profitability. On the other hand, it creates the need for new line planning after some time as significant changes occur in travel demands (Caprara et al., 2007).

The companies active in the field of passenger transportation are facing different levels of planning problems. These levels are demand forecasting, line planning, train timetabling, rolling stock planning and crew scheduling (Goossens et al., 2006).

The first step in railway transportation planning in the field of passenger transportation is demand forecasting, which is one of the prerequisites for line planning or determination of passenger route. The demands between each pair of stations are estimated in this phase. The precise estimation of travel demand is the basis for further planning efforts of passenger transportation. There are several researches regarding the estimation of passenger demands (Tasi et al., 2009; Suryani et al., 2010).

The next phase is to plan the passenger lines in which the optimal lines with known frequencies are determined. Among the most important articles in the field of line planning, we can mention Bussieck et al. (1996) which search for a line system with a maximum number of direct passengers. They use decision variables denoting the frequency of each line and assuming that all trains have the same fixed capacity. In order to reduce the number of decision variables, they use an aggregation of the decision variables. This aggregation requires the capacity constraints of the trains to be relaxed. The model is solved by first applying several preprocessing techniques, and then by applying a general-purpose integer linear programming (ILP) solver. The authors describe several valid inequalities to improve the linear programming (LP) lower bound. Claessens et al. (1998) study the problem of finding a minimum cost line system. They start with a nonlinear mixed integer model involving binary decision variables for the selection of the lines and additional variables for the frequencies and the train lengths. Since the nonlinearity of the model leads to computational problems, the authors switch to an ILP. A general-purpose ILP solver solves the model after applying several preprocessing techniques.

Goossens et al. (2004) focus on the design of a minimum cost line system, and a branch-and-cut approach to solve LPP is proposed. The main ingredients of the algorithm are preprocessing techniques combined with several classes of valid inequalities to improve the LP lower bounds and a number of strategies for variable selection and branching during the branching process. Lindner (2000) studies the minimization of the costs of the line system. He develops a branch-and-bound algorithm for finding a cost-optimal line system. His model also integrates the minimum cost LPP with a model for finding a cyclic timetable. Scholl (2005) studies the minimization of number of the passengers who need to change their trains to get to their destination and showed that it is more complicated to minimize the number of these passengers than to maximize the number of direct passengers. He used the Lagrangian relaxation method to find the lower limit.

Yaghini et al. (2012a) propose a new goal programming technique to handle two objectives of operating cost and the number of direct passengers travel by train. They consider different types of trains for public transportation of passengers in order to make the proposed model more realistic. In a following paper, Yaghini et al. (2012b) uses a column generation method to decompose a large-scale railroad passenger-scheduling problem into some smaller scale problems, which are easier to solve.

Peng, H. and Y. Zhu (2013) propose a multiple objective programming model considering maximum railway department revenue, the smallest passenger total expenditure and the maximum passenger demand. They also use the genetic algorithm to solve the established multi-objective 0-1 integer-programming model. Schöbel (2012) surveys the recent line planning models and their solution methods.

The output for the line planning provides the input for train timetabling. Train timetabling or train scheduling determines the timetable of departure and arrival of the trains to each station. Due to special importance of scheduling, several studies there have been published in this field (Burdett and Kozan, 2009; Liu and Kosan, 2009; Lee and Chen, 2009; Chung et al., 2009; D'Ariano et al., 2007).

The timetables are considered as the input for rolling stock planning. In this phase, the fleet is assigned to each train according to train timetables. Among the articles in the rolling stock planning field, we can mention (Peeters and Kroon, 2008; Ghosairi and Ghannadpour, 2010). The last phase is crew scheduling. In this phase, the train crew work plans are determined in such a way to minimize the cost of crew assignment without violating the labor legislations. See the reference (Mesquita and Paias, 2008) for more details in crew scheduling field.

In this article, a new solution method based on decomposition algorithm has been proposed, which is defined in a master problem and two sub problems. Since the solution provided by decomposition algorithm is not an integer feasible solution, a heuristic algorithm has been proposed for converting the results to integer numbers.

The mathematical model of the passenger line planning used in this study is introduced in Section 2. In Section 3, the proposed algorithm is presented and the validity of algorithm is tested by sixteen samples. In Section 4, line-planning problem in Iran Railways is solved by the proposed algorithm and the results of which and further research areas are presented in Section 5.

2. The Line Planning Problem

In this study, the line-planning model maximizes the number of direct passengers between all the starting points and destinations. The constraints of the line-planning problem include demand constraints, capacity constraints and the constraints of total number of frequencies on each arc in the network (Bussieck et al., 1996). In addition, the model can consider different types of trains with varying capacities.

The notations of the model can be described as here below.

- Indexes:

e: Arc index, $e \in E$.

r: Line index, $r \in R$.

a, b: Starting point and destination index, $a, b \in V$.

h: Type of trains or rolling stock index, $h \in G$.

- Sets:

V: Set of network stations.

E: Set of network arcs.

R: Set of suggested lines.

$R_{a,b}$: Set of lines that can move the demand of *a* and *b* directly.

G: Set of trains or rolling stocks.

- Parameters:

$d_{a,b}$: Travel demand between stations *a* and *b*.

$u(e)$: Upper bound of frequency on arc *e*.

c^h: Train or rolling stock capacity of type *h*.

- Decision variables:

$y_{r,a,b}$: An integer variable which determines the number of direct passenger transported from station *a* to station *b* by line *r*.

x_r: An integer variable which determines the frequency or departure of line *r* by rolling stock type *h*.

The mathematical formulation of the line-planning model is defined as follows.

$$Max \quad \sum_{a,b \in V} \sum_{e \in r | r \in R_{a,b}} y_{r,a,b} \qquad (1)$$

Subject to

$$\sum_{e \in r | r \in R_{a,b}} y_{r,a,b} \leq d_{a,b} \qquad \forall a,b \in V \qquad (2)$$

$$\sum_{e \in r | r \in R_{a,b}} y_{r,a,b} \leq c \; x_r \qquad \forall r \in R, e \in r \qquad (3)$$

$$\sum_{e \in r | r \in R_{a,b}} x_r \leq u(e) \qquad \forall e \in E \qquad (4)$$

$$x_r, y_{r,a,b} \geq 0 \; \& \; Integer. \qquad \forall r \in R, \forall a,b \in V \qquad (5)$$

Objective function (1) maximizes the total number of direct passengers transported from their starting point to destination on all the lines. Constraint (2) states that total number of all the passengers traveling directly from their starting point to their destination should be less than total demand for them. Constraint (3) states that for each line, the total number of direct passengers passing through a certain arc should be less than the created capacity by all types of trains departing in that line. Constraint (4) states that total number of frequencies on all the lines of each arc should be lower than the upper bound of departures on that arc. This upper bound is determined by considering railway constraints such as signaling system, which is considered as one of the input parameters in this problem. Constraint (5) defines the type of decision variables of the problem, which are integer numbers.

3. The proposed solution method

The decomposition algorithm is an exact algorithm for finding the optimal solution of the linear programming problems. This algorithm is used for solving large-scale problems for which the solutions cannot be obtained from optimization software in an acceptable time. A variety of articles are concerned with decomposition algorithms including (Bronmo et al., 2010; Oppen et al., 2010; Ronnberg and Larsson, 2010; Dezso et al., 2010).

In the decomposition algorithm, the problem is divided into one master problem and at least one sub problem.

The objective function of master problem defines the problem's objective function, and sub problems' objective functions define the result of optimal conditions. The output for master problem is the dual values, and the outputs for sub problems are selected variables and their values to append to the master problem. There is no guarantee for obtaining integer numbers for decision variables.

3.1. The proposed decomposition algorithm

For solving the line-planning problem by decomposition algorithm in the proposed solution method, constraint (3) – which includes both direct passenger number and number of line departures – is considered in the master problem, and other constraints are applied by constructing two sub problems. The first sub problem consists of constraint (1) and the second one consists of constraint (4). Since the problem solution space is divided into three partitions by one master problem and two sub problems, solving the problem is easier. The dual values are inserted from master problem into sub problems on each step, and then decision variables and their assigned values in sub problems are transferred to the master problem.

3.1.1. The master problem

As stated before, the problem is divided into one master problem and two sub problems by the proposed algorithm. The master problem is presented in Eqs. (6) to (10).

$$Min \ \sum_j (-\sum_{a,b\in V} \sum_{e\in r|r\in R_{a,b}} y_{r,a,b})\lambda_j \qquad (6)$$

$$\sum_j ((\sum_{e\in r|r\in R_{a,b}} y_{r,a,b})\lambda_j + (-c\,x_r)\gamma_j) + s = 0 \qquad \forall r\in R, e\in r \qquad (7)$$

$$\sum_j \lambda_j = 1 \qquad (8)$$

$$\sum_j \gamma_j = 1 \qquad (9)$$

$$\lambda_j \ge 0, \quad \gamma_j \ge 0, \quad s \ge 0. \qquad (10)$$

Objective function (6) is the same as objective function (1) which is minimized to comply with decomposition algorithm. In addition, *l* are the decision variables in the model. Constraint (7) is the same as constraint (3), and vector *S* defines the slack variables. Constraint (8) is the convexity constraint for variables *y* and constraint (9) is the convexity constraint for variables *x*. constraint (10) defines the decision variables in the model as positive. Vectors *x* and *y* in model (6) – (10) are the inputs from sub problems and are considered as model parameters. The dual value vector for constraint (3), (8) and (9) are shown by *w*, *α* and *β*, respectively.

3.1.2. The first sub problem

The first sub problem consists of constraint (2) and determines the values of variables *y* on each step. It is defined in the model stated by (11) – (13).

$$Max \ \ ((w\times A)+1)y+\alpha \qquad (11)$$

$$\sum_{e\in r|r\in R_{a,b}} y_{r,a,b} \le d_{a,b} \qquad \forall a,b\in V \qquad (12)$$

$$y \ge 0. \qquad (13)$$

The objective function (11) determines the decision variables of type *y* and their values for master problem. As mentioned before, *w* represents the dual value vector for constraint (7) and *A* is the coefficient for variables *y* in constraint (7). Constraint (12) is the same as constraint (2). Constraint (13) defines the decision variables for the first sub problem. The decision variables in line planning problem are of integer type, but since the solution in the decomposition algorithm has to be convergent, the variables of type *y* in the first sub problem are defined as continuous numbers.

3.1.3. The second sub problem

The second sub problem which selects the variables of type x and assigns values to them is defined as the model (14) – (16).

$$Max \quad (w\times A)x+\beta \tag{14}$$

$$\sum_{e\in r | r\in R_{a,b}} x_r \le u(e) \qquad \forall e\in E \tag{15}$$

$$x\ge 0. \tag{16}$$

In addition, the second sub problem deals with the constraints only including *x* variables. The objective function shown by (14) determines the decision variables of type *x* and their values for master problem. The constraint (15) is the same as (4), and constraint (16) defines the variables as positive. The variables of type *x* in the line planning problem, like variable *y*, is of integer type which is formed as constraint (16) due to reasons discussed at the first sub problem, but it must be converted to integer numbers.

3.1.4. Termination condition

In the line-planning problem, since there are two sub problems defined, the termination condition is given as in statement (17).

The proposed algorithm is terminated when the objective function values of both sub problems are non-positive. The obtained solution found by the decomposition algorithm enters into the proposed heuristic algorithm.

3.2. The proposed heuristic algorithm

As stated, the solution obtained from the decomposition algorithm is not necessarily an integer feasible solution. The main output for the line-planning problem is the number of train departures on each line. Therefore, a non-integer value for this variable is meaningless. A heuristic algorithm is proposed to convert the obtained results to integer numbers. The stages of the proposed heuristic algorithm are as follows.

Stage 1: The summation of frequencies on each arc (*SF*) is calculated.
Stage 2: The summation of frequencies on each arc is rounded to the nearest integer number (*RSF*).
Stage 3: Each frequency is rounded (*RF*) and their summation (*SRF*) is calculated.
Stage 4: The difference between *SRF* and *RSF* (*DSR*) is calculated.
Stage 5: According to the *DSR*, the frequency of longest possible line (lines) is added.
The proposed solution algorithm is shown by Fig. 1.

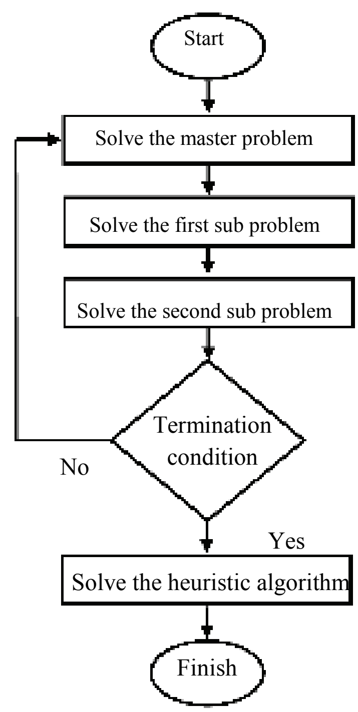

Fig 1. Flowchart of the proposed solution algorithm

$$(Max \quad (w\times A)x+\beta)\le 0 \qquad \& \qquad (Max \quad ((w\times A)+1)y+\alpha)\le 0. \tag{17}$$

The proposed solution method is executed on GAMS programming and optimization software. According to Table 1, sixteen typical problems of different sizes are generated to verify the proposed methods. In order to validate the proposed solution method, the problems are solved by CPLEX optimization software by taking into account the integer inequalities and the proposed solution method. The obtained results and solution times are compared to each other in the table. The number of stations is considered 5, 10, 15 and 20. The proposed lines are also defined in two different types. In the first scheme, the proposed lines between all the stations are defined as shown by label F. In the second scheme, half the number of the proposed lines in first scheme is defined as the proposed lines. Of course, these lines are such selected that it is possible to transport all the passengers directly, so the long lines are more focused on. The proposed lines in this scheme are shown by label H. In addition, two schemes are set for the capacity of total number of frequencies on each arc of network (u(e)). The capacity is considered as low for the first scheme (T), and high for the second one (L). Two different types of trains are considered with a capacity of 100 and 120. The demand between each two stations is 50. First column in Table 1 shows the problem number. Second column shows the number of stations, and the third one states how the proposed lines are defined. The fourth column shows the definition scheme for frequency capacity on each arc of network. The fifth column shows the optimum objective function value obtained from CPLEX optimization software considering integer inequality using branch-and-bound algorithm. The sixth column is the time needed for obtaining optimum solution by the optimization software. The seventh column includes the optimum objective function values obtained from the proposed solution method, and finally the eighth column shows the required time for obtaining optimum solution with the proposed algorithm. The last column in Table 1 shows the ratio of total number of direct passengers to total demand. For example, problem 1 considers 5 stations and definition of all possible lines (F) and low capacity for number of frequencies on each network arc. It yields 330 as optimum objective function obtained from CPLEX optimization software in 1.2 seconds, while the optimum objective function obtained from the proposed algorithm is 330 in 1.3 seconds. For problems 13 and 14, CPLEX cannot obtain optimal solutions in less than 7200 seconds, but the optimal solution for these problems are obtained from the proposed solution method in a suitable time.

From Table 1, it is concluded that the proposed solution method generates optimal solutions for test problems in an appropriate time. Therefore, this algorithm can be used for real world problems like those that Iran Railways network to optimize and improve the passenger lines.

4. Case study

In this section, the passenger line-planning problem in railway network of Iran is optimized by considering the important stations serving more passengers. Out of all active passenger stations in Iran, 23 of them serving more passengers are assumed to be connected by 29 network arcs. Fig. 2 shows the Iranian railway network. Stations in Fig. 3 are used for this case study and Table 2 shows the catchment populations.

175 potential lines are defined for Iranian railway network as shown in Fig. 3. The planning period is one week. That means the travel demand between the different stations are stated for one week and the obtained number of departures on each line is for one week. Travel demand between all the starting and destination stations in Fig. 5 for one week equals to 454,000 passengers, and one type of train is considered with a capacity of 500 passengers. The input data for case study is given in Table 3.

The proposed algorithm and GAMS software solve the model of Iranian railway passenger lines. The optimal solutions obtained are shown in Table 4. According to the the table, 41 lines are selected as optimal lines from 175 proposed lines. These lines transport the maximum number of direct passengers in Iran Railways network. For example, second row of Table 3 states that Tehran–Gorgan line is one of the selected lines of the passenger lines which has an optimal number of 14 departures per week.

5. Conclusion

In this paper, a new solution method based on decomposition algorithm is presented for passenger

Table 1. Solved problems by CPLEX and the proposed algorithm

P.N	N.S	N.L	A.C	CPLEX		The proposed solution method		Rate
				O.F	S.T (s)	O.F	S.T (s)	Num. direct passenger/total demand
1	5	F	T	360	1.6	360	1.7	0.72
2	5	F	L	500	1.3	500	1.5	1
3	5	H	T	360	0.8	360	1.0	0.72
4	5	H	L	500	0.6	500	0.9	1
5	10	F	T	1610	448.2	1610	74.1	0.71
6	10	F	L	2250	402.7	2250	65.2	1
7	10	H	T	1610	50.4	1610	12.6	0.71
8	10	H	L	2250	46.3	2250	11.9	1
9	15	F	T	3690	7157.9	3690	771.9	0.7
10	15	F	L	5250	6692.8	5250	706.8	1
11	15	H	T	3690	867.1	3690	124.1	0.7
12	15	H	L	5250	788.2	5250	108.3	1
13	20	F	T	-	-	6940	2395.8	0.73
14	20	F	L	-	-	9500	2196.7	1
15	20	H	T	6940	6903.1	6940	603.3	0.73
16	20	H	L	9500	6221.9	9500	527.5	1

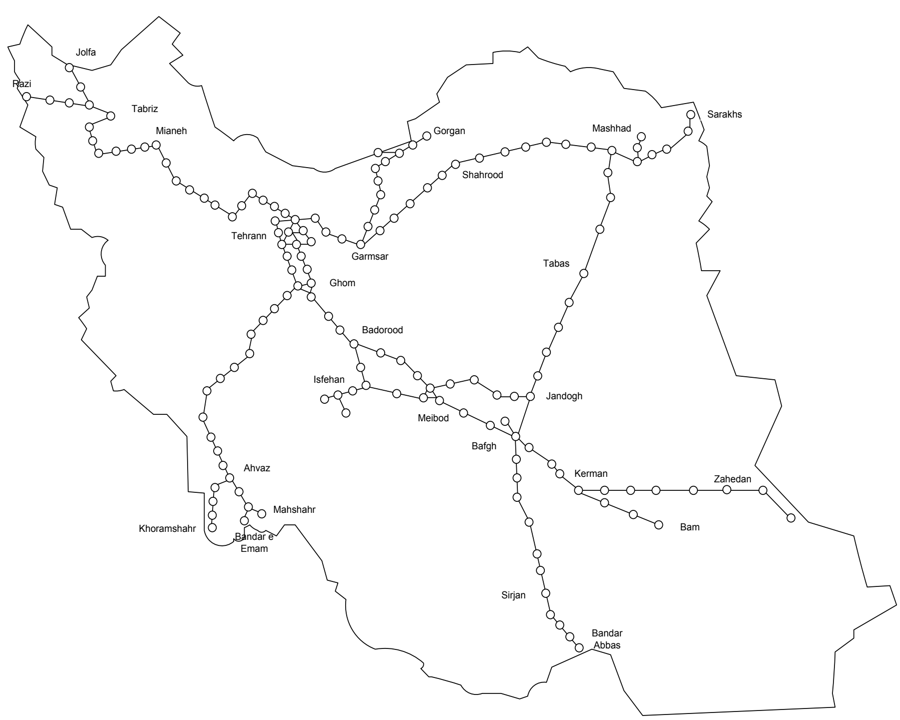

Fig 2. Iranian Railways network.

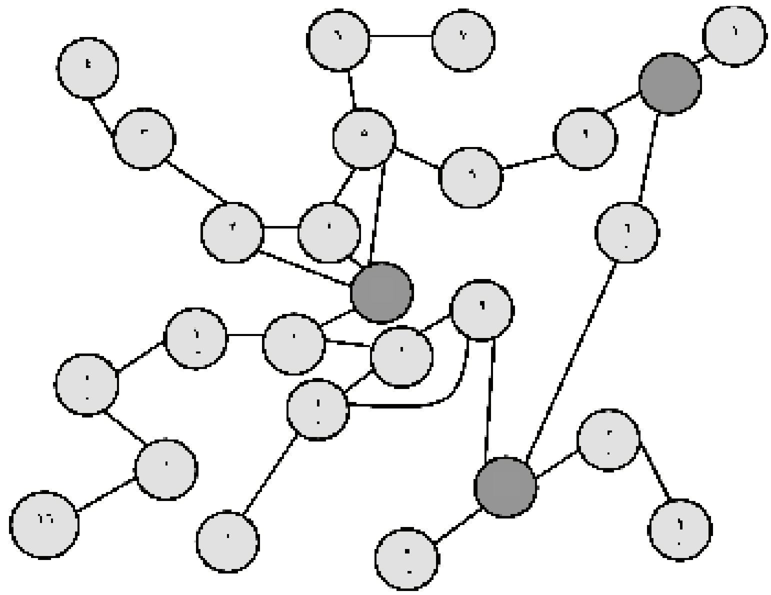

Fig 3. Considered stations in case study.

Table 2 . Catchment populations

Number of station	station	Population*	Number of station	station	Population*	Number of station	station	Population*
1	Tehran	7.705.036	9	Shah rood	126.916	17	Kashan	248.789
2	Karaj	1.377.450	10	Mashhad	2,410.800	18	Esfahan	1,583.609
3	Zanjan	341.801	11	Tabas	30.681	19	Shiraz	1,214.808
4	Tabriz	1.378.935	12	Qom	951.918	20	Yazd	423.006
5	Garmsar	38.891	13	Arak	438.338	21	Kerman	496.684
6	Sari	259.084	14	Andimeshk	119.422	22	Zahedan	552.706
7	Gorgan	269.226	15	Ahvaz	969.843	23	Bandar e abbas	367.508
8	Semnan	124.999	16	Khoramshahr	123.866			

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Table 3. Input data for case study

Total demand per week	Number of stations	Total potential lines	Capacity of each train
454000	23	175	500

Table 4. Selected lines and their frequencies per week

	Selected line	Frequency		Selected line	Frequency
1	Tehran-Sari	11	22	Mashhad-Yazd	33
2	Tehran- Gorgan	14	23	Mashhad-Bandar e abbas	7
3	Tehran-Zanjan	21	24	Mashhad-Kashan	10
4	Tehran-Tabriz	26	25	Mashhad-Esfahan	22
5	Tehran-Shahrood	12	26	Mashhad-Shiraz	16
6	Tehran-Mashhad	113	27	Mashhad-Qom	20
7	Tehran-Tabas	8	28	Mashhad-Zanjan	19
8	Tehran-Arak	16	29	Mashhad-Tabriz	25
9	Tehran-Andimeshk	10	30	Mashhad-Arak	16
10	Tehran-Ahvaz	39	31	Mashhad-Ahvaz	20
11	Tehran-Khoramshahr	9	32	Mashhad-Khoramshahr	12
12	Tehran-Kashan	12	33	Tabriz-Karaj	28
13	Tehran-Esfahan	23	34	Kerman-Tabas	6
14	Tehran-Shiraz	15	35	Kerman-Bandar e abbas	10
15	Tehran-Yazd	20	36	Yazd-zahedan	21
16	Tehran-Kerman	26	37	Esfahan-Bandar e abbas	19
17	Tehran-Zahedan	10	38	Esfahan-Shiraz	16
18	Tehran-Bandar e abbas	24	39	Arak-Ahvaz	17
19	Mashhad-Semnan	32	40	Andimeshk-Ahvaz	18
20	Mashhad-Gorgan	25	41	Ahvaz-Khoramshahr	18
21	Mashhad-Zahedan	12			

line planning problem. The line-planning model used in this paper includes the objective function for maximizing the number of direct passengers considering the demand constraints, capacity provided by the lines, and upper bound for frequencies on each arc of the network.

In the proposed solution method, the problem is divided into one master problem and two sub problems in this algorithm and therefore the solution space for the problem is divided into three partitions. The decomposition algorithm starts by solving the main problem. The dual values are conveyed to the sub problems after each time of solving the master problem; then the sub problems are solved and the decision variables' values are transferred to the master problem again. The algorithm continues until the objective functions of the sub problems determine the current solution is optimal.

The variables of line planning problem are of integer type, while the decomposition algorithm cannot generate integer solution. Therefore, a heuristic algorithm is proposed to convert the obtained solutions from decomposition algorithm to the integer solutions. Several typical problems are generated and solved using CPLEX optimization software and the proposed solution method, and the results and solution times are compared. The proposed algorithm obtains the optimal solutions in an appropriate time for all the typical problems.

As a case study, Iran Railways network is examined considering 23 stations with high number of passengers, and defining 175 potential passenger lines between the stations. 41 lines among the total 175 proposed lines are selected as optimal passenger lines for Iran railways. Using the proposed model can improve the level of passenger satisfaction and consequently increase the profitability. As a basis for future researches, the branch-and-price algorithm can be used to obtain the optimal integer solutions. In large size problems, the metaheuristic algorithms such as genetic algorithm can be used to obtain near optimal solutions in a short computational time. In addition, the use of energy sources, the effect of crowding, and using reliability concept in modeling could be considered in the future works.

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