Investigation of sloshing coefficient by Arbitrary Lagrange - Euler methods in partially filled tankers

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Abstract:
Regarding increasing importance of tank wagons and efficiency in fluids transportation, their usage is increasing every day. Thus paying attention to their designs just as dynamic analysis of wagons during travelling time is vitally important. In this task, sloshing coefficient as prominent parameter in tank wagons designing has been investigated. To ease of efficient design, theses parameters can cause accurate and economic design. For purpose of derivation of this coefficient, Arbitrary Lagrange-Euler (ALE) method was applied and results were compared with equivalent mechanical method (EM). So, to simulate fluid sloshing inside the tank two different approaches, Finite element method and multi body dynamics were exploited. In lateral and longitudinal simulations, tanker is analyzed in traveling curved track and accelerating respectively. Also, the effects of velocity, irregularities of track and fill ratio on sloshing coefficient were studied. These results are proposed to be used in tank design and related manuals.

Keywords: Tank Wagon- Sloshing- Multi body dynamics - ALE method
1- Introduction

Increasing importance of fluids trade and transport by usage of ground transportation and intensity increase of transportation needs more attention in related section. Most of research studies explored dynamic behavior of moving partially-full tank wagon, although enough attention haven’t been paid to these design cases. To this day, studies in the field of sloshing are done by using continuous and discontinuous methods separately. [Aliabadi, Johnson, Abedi, 2003] have discussed the longitudinal and lateral forces caused by fluid motion in a truck tank and compared results in both pendulum and finite element methods. Both approaches have the same behavior, but the force amplitude of the pendulum method is a bit more than the continuous method.

Reference [Feizi, Sajjadi and Shahravi, 2014] has investigated fluid behavior in tank wagons by Euler approach. Using finite element, the behavior and profile of fluid are estimated by 2 dimensional modeling and have been evaluated with test results. The train body was considered rigid and pressure distribution and fluid velocity were studied. [Younesian, Abedi and Hazrati Ashtiani, 2010] have studied lateral sloshing by mechanical equivalent model approach and mass-spring method. In this task, train stability in various curve radiuses and fill ratio percentages has been studied and derailment quotient and unloading ratio have been calculated. The stability of tanker trucks, longitudinal models have received less consideration [Ervin, 1985]. This issue finds special importance in train’s arrangement. Studies in this field have been carried out mostly by using multi-body dynamics and using dynamic equivalent models. [Vera, Paulin, Sua’rez, and Gutie’rrez, 2005] investigated the stability of train by mechanical equivalent model approach. In this study, pendulum and spring-mass approaches have been used in modeling of lateral and longitudinal stability and by use of this modeling; forces between couplers have been extracted. Sloshing in tankers during braking was analyzed by applying 3-dimensional finite element [Aliabadi, Johnson, Abedi, 2003]. While the tanker is travelling by constant velocity of 10 m/s, 0.2 g brake acceleration is applied. Lateral sloshing has been compared by applying mechanical and finite element approaches. Results are in suitable accordance, but force amplitude in pendulum method (mechanical approach) is larger than finite element method.

As noted, sloshing coefficients in rail transport tank design are vitally important; nevertheless no comprehensive study has been carried out. In simulation of these problems, continuous methods appear to be more precise, while being time consumer prevents it to be perfect solving method. Through this paper, applying ALE approach is suggested for investigating sloshing in tank wagon and sloshing coefficient is extracted by calculated force in various scenarios of travelling. The results of this approach have been compared with mechanical approach.

2- Sloshing coefficient in ASME standard (Lateral and longitudinal coefficient)

In section TD-2 of ASME standard, named as loading and allowable stress, it is mentioned that the forces of liquid sloshing in the tanker should be considered as an effective forces during preliminary design. Consequently, to define the sloshing force, a static equivalent vector is defined. This vector is comprised of sloshing coefficients, $C_S$, multiplied by weight of fluid contents of the tank and applies to the tanker’s wall perpendicularly in both longitudinal and lateral directions:

$$F_s(L_0) = C_s(L_0) \cdot W$$  \hspace{1cm} (Eq. 1)

$$F_s(L_a) = C_s(L_a) \cdot W$$  \hspace{1cm} (Eq. 2)

In above relations, $L_0$ and $L_a$ are longitudinal and lateral directions respectively. $F_z$, $F_y$, $F_x$ forces have been defined in this standard separately [ASME, 2011]. Figure 1 is a presentation of forces in various directions against the tank wall.

![Figure 1: Tanker forces in three directions](image)

3-Finite element modeling method

3.1. Various approaches for continuous simulation

In general, finite element approaches for modeling of
sloshing phenomena are divided into 4 approaches: Euler, Lagrange - Euler, SPH, and Lagrange.

3.1.1 Lagrange approach
This approach usually is used for describing solid mechanic problems or problems that all their particles are known. Lagrange approach is very easy for modeling one or more separated masses but it is very difficult for describing many particles. In this approach, boundary of problem is defined as well. This approach is suggested for problems including little deformation.

3.1.2 Euler approach
This approach is suitable for models with large deformation which its elements mesh remains unchanged and physical materials are transferred by these elements. Main difference between Lagrange and Euler is that in Lagrange approach, x, y, z for moving mass is variable. Whereas in Euler approach, coordinates of each point from mesh is constant. Thus, this approach is suitable efficient in liquids and gas behavior modeling.

3.1.3 SPH approach
This approach is united plan which is based on Lagrange approach and it has been developed in order to avoid any mesh limitation when deformation is large. Main difference between SPH and standard approach is lack of mesh in SPH method.

3.1.4 Arbitrary Lagrange - Euler approach (ALE)
This method includes both Lagrange and Euler models. In this method, while mesh is deforming and moving (Lagrange approach), the material flow (Euler approach). In tank analysis, when fluid movement inside it is under consideration and boundary surface changes continuously because of interaction between fluid and tanker, mesh will be modified automatically. It’s one of advantages of ALE method.

3.2 longitudinal models in finite element approach
In ALE approach modeling, the tank wall is considered elastic, while in Euler approach, tank wall considered rigid in order to increase the calculation speed. With this assumption in arbitrary Lagrange - Euler approach, condition is prepared for inspecting the tank wall state and forces. Air and liquid inside tank are modeled by solid element. All braking and accelerating path must be defined by air element. In this simulation, Parsi wagon with length of 11.6 and diameter of 2.6 meter has been selected for modeling. Totally, 6912 shell elements and 91526 solid elements are used in longitudinal modeling of this wagon. In order to accelerate the calculation; thin layer of tank has been analyzed. Figure 2 depicts longitudinal model, air and liquid elements and tank wall.

3.3 Lateral model in finite element approach
In this section, in according to previous modeling, thin strip of tank is considered. All length of curve has been modeled by air elements. Totally 1728 shell elements and 356080 solid elements constitute the lateral model.

4. Equivalent mechanical modeling method
4.1. Y25 Bogie modeling
This bogie has built by France railway in about 1960 and according to Y21A. In this bogie, stiffness is load-dependent and modeled by load-dependent damping by using Lenior-link element in ADAMS/RAIL software [Molatefi, Hecht and Kadivar, 2006]. This bogie is illustrated in fig. 4.

Mass values, locations, center of mass and moment of inertia are presented in table 1. In this modeling, all dry frictions are modeled by Kolsch method and impacts in clearance are modeled by non-linear spring.

Primary suspension forces
In Y25, clearance between axle box and bogie frame in vertical direction is about 55 mm and in horizontal direction is +/-10 mm and in longitudinal direction is 4 mm in
one direction. Impact between various parts after clearance, has been modeled by non-linear springs with maximum stiffness of $10^7$ N/m according to figure 5.

**Figure 4:** Y25 bogie. (a) lenior link (b) damping (c) sideview [Younesian, Abedi and Hazrati Ashtiani, 2010]

**Figure 5:** Nonlinear spring in longitudinal direction of primary suspension [Molatefi, Hecht and Kadivar, 2006]

Primary suspension system, according to the test which has been carried out in TU-Berlin [Keudel, 2003], is considered in shape of Hysteresis loop in 3 directions for different pre-load. Figure 6 shows the hysteresis loop in vertical direction.

For modeling of primary suspension force element, the Kolsch [Kolsch, 2003] element method is used which is presented in equation 3. In this equation, $m$ is equal to sharpness of the transition region between slip and stick shown in figure 7.

\[
C_0 = C_n - C_g
\]

\[
\dot{K} = C_0 \dot{X} \left( 1 - 0.5(\text{sign} (\dot{X}K) + 1) \right) \frac{|2K|}{F_D}^m
\]

\[
F = C_g X + K
\]

**Figure 6:** Force-displacement diagram in vertical direction [Molatefi, Hecht and Kadivar, 2006]

In figure 7, displacement-force diagram in vertical direction is shown for $m=10$.

Forces between wagon and bogie

Bogie frame is connected to wagon by central pivot and side bearers. Central pivot which is a spherical connection shown in figure 8.

The bogie frame has angular motions about the joint centre with respect to the carbody. Contact surfaces and their proportional movements make dry friction in central pivot. Researches and tests done by Nielsen [Nielsen, 1996] show that moment of central pivot has its maximum value and which is calculated by equation 4.

\[
M_x = M_y = 0.17 \cdot \mu_{\text{CentPiv}} \cdot F_N
\]

\[
M_z = 0.18 \cdot \mu_{\text{CentPiv}} \cdot F_N
\]

**Figure 7:** Force-displacement diagram in vertical direction [Molatefi, Hecht and Kadivar, 2006]
These values are for slip mode and in stick mode which angular stiffness is about $10^7$ Nm/Rad, dry friction coefficient is considered as 0.22.[Jendel, 1997]

In figure 9, side bearer with spring load is shown. Stiffness value in vertical direction is equal to $5.7 \times 10^4$ N/m and preload value is 16 KN and clearance play for spring is 12 mm. This value is approximately equal to 0.014 rad related to roll movement of wagon and bogie. Hence, Minimum force on the side bearer is 9160 N and exceeding the force of 22840 N, solid contacts occur. For damping in vertical direction, 1000 Ns/m has been used [Jendel, 1997]. dry friction between the side bearer and the wagon has been modeled with the value of 0.22 for dry friction coefficient and $10^6$ N/m for stick stiffness.

4.2 Rail and wheel profiles

In this task, the modeling of wheel and rail is carried out ADAMS/Rail software, the wheel has been considered as S1002 type and the rail as the standard rail of UIC60. In figure 10, wheel and rail profile are shown:

4.3. Lateral modeling

In section of mechanical model, both spring-mass and pendulum methods are used. Studies show, for tanks with round walls affected by lateral excitation, pendulum model presents better results in fluid slosh modeling [Sumner, 1965]. In equivalent model, fluid mass is divided in two parts of moving and fix mass which are simulated by a pendulum and a fixed mass respectively [Abramson, 1996].

Table 1: Specification of Y25 Bogie

<table>
<thead>
<tr>
<th>Y25 Bogie</th>
<th>Centre of mass (m)</th>
<th>Moment of inertia (kg m²)</th>
<th>Mass (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheelset with axle box</td>
<td>$h_{wb} = 0.46$</td>
<td>902 108 906</td>
<td>1380</td>
</tr>
<tr>
<td>Bogie</td>
<td>$h_b = 0.61$</td>
<td>1188 1484 2582</td>
<td>1990</td>
</tr>
</tbody>
</table>

* Mass center is considered from rail
Fig. 11, presents parameters of pendulum method. It is considerable that vibration mode has effect on model parameters and number of pendulums for modeling of sloshing is equal to dominant mode of fluid movement. Since in exciting of tank wagon, first mode is dominant, this mode is selected as vibration mode; as a result, one pendulum is selected for modeling.

In following figure, \( h \) is fluid height from the tank base, \( R \) is tank radius, \( l_p \) is pendulum length, \( m_f \) is fixed mass, \( m_p \) is pendulum mass and \( h_0 \) is height of fixed mass location from tank base.

Finally, set of 2 bogies and a wagon body including one pendulum are assembled in ADAMS/Rail software (Figure 16).

For longitudinal sloshing modeling (separated from lateral model), mass and spring approach is applied. In this case, sprung mass plays the role of moving part of contained fluid inside the tank. Dominant vibration mode in longitudinal state is one, thus one moving mass has been considered for modeling. Figure 13, is a schematic view of one cubic tank with equivalent mass-spring model. Equations 5-10, present the calculation method of the shown parameters in fig. 13.

\[
\begin{align*}
    m_f &= \rho \cdot h \cdot w \cdot b \\
m_1 &= m_f \cdot \left( \frac{w}{387 \cdot h} \right) \cdot \tanh (3.14 \cdot \frac{h_0}{w}) \\
m_+ &= m_f - m_0 \\
k_1 &= m_f \cdot \left( \frac{g}{1.2 \cdot h} \right) \cdot \left( \tanh \left( 3.14 \cdot \frac{h_0}{w} \right) \right)^2 \\
l_1 &= \frac{w}{1.57} \cdot \tanh (1.57 \cdot \frac{h}{w}) \\
l_2 &= \frac{h}{2} \cdot \frac{m_+}{m_0} \left( \frac{h_0}{2} - l_1 \right)
\end{align*}
\]

Just like lateral section, the same bogies are used for analyzing in longitudinal direction, except that the wagon includes mass and spring and belongs to longitudinal analysis. Fig. 17 shows assembled longitudinal model including springs and masses.

5- Expressing primary conditions
In order to analyze in lateral direction, fluid sloshing coefficient is calculated during the travel of train on a curve track and for fulfillment of longitudinal analysis, fluid sloshing coefficient is calculated during acceleration and brake situations. Mentioned states are most critical state for each of longitudinal and lateral movement. Meanwhile among the whole analysis, track irregularities are considered.
In Multi body Dynamic solution, the primary condition of problem, are completely defined in ADAMS software, but in finite element software, at first, wagon displacement over track is calculated, and then applied to tank body elements.

5.1. Track irregularity

Based on random nature of track irregularity, for presenting them, power spectral density functions are used [Frýba, 1996]. FRA (Federal rail-road administration, USA) classifies Track irregularity from class 1 to class 6. Following equations are representative for PSD functions developed by FRA that are representatives for cross and gauge, and elevation and alignment [Frýba, 1996], [Apiwan, Kuang-Han, Vijay, 1982]:

\[
S_x(\Omega) = \frac{A\Omega_1^2(\Omega_1^2 + \Omega_2^2)}{(\Omega_1^2 + \Omega_2^2)\Omega_1^2 + \Omega_2^2)}
\]  

Eq. (11)

\[
S_z(\Omega) = \frac{A\Omega_2^2(\Omega_1^2 + \Omega_2^2)}{\Omega_1^2(\Omega_1^2 + \Omega_2^2)}
\]  

Eq. (12)

In equations 11 and 12, PSD are irregularities of rail based on wave length \(\Omega_1, \Omega_2\), \(A\) are stable coefficients that their amounts are presented in table 2.

For calculating Track irregularity based on place of \(r(x)\), Au’s approach would be applied. [Tat Kwong, 2002]

\[
r(x) = \sum_{k=1}^{N} a_k \cos (\omega_k x + \phi_k)
\]  

Eq. (13)

In equation 13, \(a_k\) is \(\omega_k\) amplitude, is frequency between \([\omega_1, \omega_2]\) which calculates domain and \(\phi_k\) is a random value with normal distribution in \([0, 2\pi]\). Also, \(x\) is position on track and \(N\) shows the number of dominant segments. \(a_k\) and \(\omega_k\) are calculated by following method:

\[
a_k = 2\sqrt{S(\omega_k)\Delta \omega} \quad k = 1, 2, ..., N
\]  

Eq. (14)

\[
\omega_k = \omega_1 + (k - 1/2)\Delta \omega \quad k = 1, 2, ..., N
\]  

Eq. (15)

\[
\omega_k = \omega_1 + (k - 1/2)\Delta \omega \quad k = 1, 2, ..., N
\]  

Eq. (16)

Figures 14 and 15 show vertical and horizontal irregularities for classes 1, 3, 6 respectively.

<table>
<thead>
<tr>
<th>Irregularity</th>
<th>Parameter</th>
<th>Track class</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Notation</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Unit</td>
<td></td>
</tr>
<tr>
<td>Elevation</td>
<td>A</td>
<td>10^8 m^3</td>
</tr>
<tr>
<td></td>
<td>(\Omega_1)</td>
<td>10^3 m^-1</td>
</tr>
<tr>
<td></td>
<td>(\Omega_2)</td>
<td>10^3 m^-1</td>
</tr>
<tr>
<td>Gauge</td>
<td>A</td>
<td>10^8 m^3</td>
</tr>
<tr>
<td></td>
<td>(\Omega_1)</td>
<td>10^3 m^-1</td>
</tr>
<tr>
<td></td>
<td>(\Omega_2)</td>
<td>10^3 m^-1</td>
</tr>
</tbody>
</table>
5.2. Geometrical specifications of track
In case of longitudinal analysis, the track is tangent track, but in lateral state, the passes are curved with specifications presented in table 3:

6-results
As mentioned before, firstly, fluid behaviors of both approaches are considered. Figure 16, shows lateral view of tank wagon moving in curve track, which is extracted using 2 approaches of finite element (Arbitrary Lagrange-Euler) and equivalent mechanical.

In both lateral and longitudinal simulations in finite element modeling, all route of wagons movement is meshed. Thus, a large amount of elements are needed for route simulation. Increasing of element, more time consumed for solution. In order to accelerate the solution, larger elements are used, and this elements expansion continues to the point that any further enlargement leads to loss the acceptable accuracy of solution. Also, presentation of geometrical movement of fluid is affected by this element expansion (Figure 16), but evaluation of fluid and solid interaction and sloshing coefficient are almost unaffected.

Figure 16: lateral views of FE and equivalent mechanical models

Since the lateral acceleration during curving is less than those occurred during braking and accelerating, variations in fluid level would be less, consequently, it is expected that lateral sloshing coefficient be less than longitudinal sloshing coefficient.

Figure 17, depicts the states of braking in both finite elements and equivalent mechanical models.

Figure 17: longitudinal views of FE and equivalent mechanical models

6.1. Investigation of curve radius variations in lateral sloshing
With assumption of % 50 fill ratio, sloshing coefficients for various velocities in three different curve radiuses are shown in figure 18. This coefficient changes the range from 0.0005 to 0.008. As it is expected, in curves with smaller radius, more sloshing coefficient is observed, and meanwhile in curves with radius of 1000 and 2000 meters (curves with large radius) in low velocity (less than 12 m/s), sloshing coefficient is independent from curve radius.

Figure 18: sloshing coefficients versus different velocities and curvature

Besides that, increment rate of sloshing coefficient, in the speeds of more than 16.5 m/s extremely decreases and would be almost fixed. Finite element and equivalent mechanical approaches shows an acceptable accordance considering in high velocity, the maximum difference doesn’t exceed 6.5%. It should be mentioned that the presented analysis in this section, have been done regarding

Table 3: curve geometrical specifications

<table>
<thead>
<tr>
<th>Radius (m)</th>
<th>Primary straight (m) track</th>
<th>transient curve(m)</th>
<th>(m) curve</th>
<th>(mm) cant</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>50</td>
<td>236</td>
<td>214</td>
<td>238</td>
</tr>
<tr>
<td>1000</td>
<td>50</td>
<td>118</td>
<td>332</td>
<td>144</td>
</tr>
<tr>
<td>2000</td>
<td>50</td>
<td>60</td>
<td>390</td>
<td>48</td>
</tr>
</tbody>
</table>
to irregularity class 6.

6.2. Investigation of effect of track irregularity class in lateral sloshing

One of the other effective parameter on sloshing coefficient is the track irregularity that is shown in figure 19. With decrement of track quality, the sloshing coefficient increases, in such a way that maximum sloshing coefficients in two classes of 1 (the worst track quality) and 6 (the best track quality), have discrepancy of more than 50%.

Also, difference of equivalent mechanical approach (pendulum method) and ALE is acceptable and shows similar trend.

![Figure 19: sloshing coefficients versus different velocities and curvature](image)

6.3 Investigation of Fill ratio effects in lateral sloshing

Effect of Fill ratio is another effective parameter in sloshing coefficient which has been shown in figure 20 for different velocity. With decrease of Fill ratio, the sloshing coefficient increases.

![Figure 20: lateral sloshing coefficients versus different velocities and fill levels](image)

6.4 Investigation of effect of velocity and Fill ratio on longitudinal sloshing during braking

During train travelling, since longitudinal accelerations are more vigorous compared to lateral accelerations, so longitudinal sloshing coefficient is very larger. Figure 21, shows sloshing coefficient trend in longitudinal direction during the brake. As it is obvious, increase of fluid level leads to increment in sloshing coefficient, and this incremental trend continue to the point that the fluid can move freely in longitudinal direction. As can be seen in figure 21, sloshing coefficient in %98 of fill ratio is approximately %2 less than sloshing coefficient in %80 which is consequence of replacement limitation in fluid in 98%. Generally, with increase of fill ratio of fluid, no significant variations occur. Difference between sloshing coefficients with fill ratio of 30% and 80% is around %13.

![Figure 21: longitudinal sloshing coefficients versus different velocities and fill levels](image)

6.5 Investigation of effect of fill ratio on longitudinal sloshing during acceleration (velocity of 25 m/s)

Figure 22 shows sloshing coefficients trend in two states of braking and acceleration. Difference of these coefficients is consequences of differences in acceleration rate in above said states. According to mentioned velocity, maximum longitudinal sloshing coefficient is less than 1.6 which shows that with assumption of 2 for sloshing coefficient for designing tank wagons, a suitable reliability has been kept.

![Figure 22: comparisons of longitudinal coefficient in decelerate and accelerate](image)
7-conclusion

Through this paper, sloshing coefficient that has an effective role in designing tank wagon has been investigated. By use of arbitrary Lagrange-Euler approach in conditions of braking, acceleration and curving, sloshing phenomena in longitudinal and lateral states has been surveyed. Also, using Monte Carlo approach, longitudinal and lateral irregularities of track were extracted and its effect is considered in analysis.

In order to verification of the results, exerting ADAMS/Rail software and equivalent mechanical approach, simulated scenarios were repeated. Finally, it was understood that the results had good similarity, and the effects of each parameter were presented as well. Besides that, Sloshing coefficient has been estimated to be between 1.2 and 1.6 in longitudinal state and up to 0.16 in lateral states.

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